On the Influence of GVR in Vehicle Routing

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ABSTRACT

A comparative study is made between a new evolutionary approach for the Vehicle Routing Problem (VRP) and a standard evolutionary model, based on Path Representation. Genetic Vehicle Representation (GVR) is the new two-level representational scheme designed to deal in an effective way with all the information needed by candidate solutions. Experimental results, obtained from a set of VRP instances, show performance improvements when GVR is used.

Keywords

Genetic algorithms, representation, vehicle routing problem

1. INTRODUCTION

The Vehicle Routing Problem (VRP) is a complex combinatorial optimization problem, which can be described as follows: given a fleet of vehicles with uniform capacity, a common depot, and several customer demands, find the set of routes with overall minimum route cost which service all the demands. All the itineraries start and end at the depot and they must be designed, in such a way, that each customer is served only once and just by one vehicle. The VRP is NP-Hard.

Due to the nature of the problem it is not viable to use exact methods for large instances of the VRP (for instances with few nodes, the branch and bound technique is well suited and gives the best possible solution [7]). Most approaches rely on heuristics that provide approximate solutions [15], [3]. Another alternative is to apply standard optimization techniques, such as tabu search [4], [11], simulated annealing [11], [1], or ant colony optimization [5].

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Applications of evolutionary computation (EC) techniques to the VRP are also used (see [2] for a brief overview). Most researchers rely on hybrid approaches that combine the power of an EC algorithm with the use of specific heuristics [14], [12], [6], or use simplified versions of the problem. One common simplification is to pre set the number of vehicles that is going to be used in the solution [16]. The first attempts to apply standard EC algorithms to the most generic version of VRP attained a limited success [4], [8].

As soon as we started our research about the application of EC techniques to the VRP, we realized that representation is a key issue. As such, in a previous work we have proposed a new representational scheme, Genetic Vehicle Representation (GVR) [10] (see also [13]), to deal efficiently with the two levels of information that a candidate solution must encode: clustering of the demands (i.e., allocation of all the demands to different vehicles) and specification of the delivery ordering for each one of the routes. GVR also enables an easy adjustment of the number of vehicles required. It is important to notice that our methodology does not use any specific heuristic. In the above mentioned work, we performed several preliminaries experiments with some well known benchmarks, that enabled us to discover new best solutions.

In this paper, a systematic study is made, with the main purpose of accessing the efficiency of the GVR approach. We present the results of a comprehensive set of tests with two variants of the VRP, and compare the performance of this approach with standard EC models.

The paper has the following structure: in section 2 we give a formal definition of the problem variants used. In section 3 we present GVR and describe the standard EC algorithms used in the tests. Section 4 comprises the presentation of the experimental results with a brief analysis. Finally, in section 5, we draw some overall conclusions.

2. THE VEHICLE ROUTING PROBLEM

2.1 Capacitated Vehicle Routing

The most general version of the VRP is the Capacitated Vehicle Routing Problem (CVRP), which can be formally described in the following way: there is one central depot

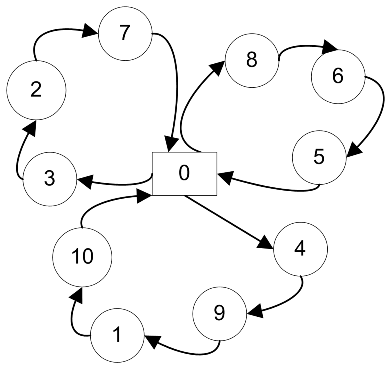

Figure 1: Vehicle Routing Problem.

0, which uses k independent delivery vehicles, with identical delivery capacity C, to service demands d_i from n customers, $i = 1,\ldots,n$. The vehicles must accomplish the delivery with a minimum total length cost, where the cost c_{ij} is the distance from customer i to customer j, with $i,j \in [1, n]$. The distance between customers is symmetric, i.e., $c_{ij} = c_{ji}$ and also $c_{ii} = 0$. A solution for the CVRP would be a partition $\{R_1,\ldots,R_q\}$ of the n demands into k routes, each route R_q satisfying

$$\sum_{p \in R_q} d_q \leq C. \tag{1}$$

Associated with each partition is a permutation of the demands belonging to it, specifying the delivery order of the vehicles. In figure 1 we present an illustration of the problem, viewed as a graph, where the nodes represent the customers.

2.2 Vehicle Routing with Time Windows

An important extension to the problem is the Vehicle Routing Problem with Time Windows (VRPTW) which adds several time constraints to the previous definition. Associated with each customer i there is a time window $[e_i,\ l_i]$ during which it has to be served. This way, a vehicle must arrive to i no sooner than e_i (the earliest arrival time) and no later than l_i (the latest arrival time). Additionally, there is a service time f_i for each customer and a limit T, defining the maximum travel time permitted for any vehicle. The travel time from customer i to customer j is $t_{ij} = c_{ij}$.

Like in the CVRP, a candidate solution would be a partition $R_1,\ldots,R_k$ of the n demands into k routes, each route R_q satisfying both the delivery capacity and the time constraints.

3. EVOLUTIONARY MODELS

3.1 Genetic Vehicle Representation

A candidate solution to an instance of the CVRP or VRPTW must specify the number of necessary vehicles, the partition of the demands through all these vehicles and also the delivery order for each route. In GVR, the genetic material of an individual contains several routes, each one of them composed by an ordered subset of the customers. All demands belonging to the problem being solved must be present in one of the routes. As an example, the chromosome from figure 2 represents the solution presented in figure 1.

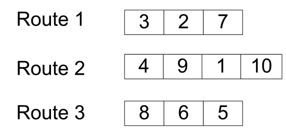

Figure 2: An example of a GVR chromossome.

The information encoded in the chromosome must be interpreted in such a way that it yields a legal solution. When a vehicle exceeds its capacity, the according route is split in several ones. An example illustrates this adjustment: assume that the original route $\{a,\ b,\ c,\ d,\ e,\ f\}$ causes the vehicle to exceed its capacity at node d. When this situation occurs, the itinerary is divided in two sections: $\{a,\ b,\ c\}$ and $\{d,\ e,\ f\}$, and a new vehicle is added to the solution. If necessary, further divisions can be made in the second section.

For the VRPTW, another specific situation arises, when time constraints are not satisfied. In this case, three different types of violation may occur: early arrival at a customer, late arrival at a customer or late arrival at the depot. The first situation is easily solved, since it's only required that the vehicle waits until it meets the earliest arrival time of the window. To resolve the other two cases, a new section is created on the itinerary, providing a valid route by adding a new vehicle to the solution. Notice that all these changes only occur at the interpretation level and, therefore, the information codified in the chromosome is not altered.

3.2 Genetic Operators

The EC algorithm processes the individuals in a straightforward way. Assuming that the population size is N, in each generation, N parents are chosen and N descendants are obtained through the application of genetic operators to the elements of the selected set.

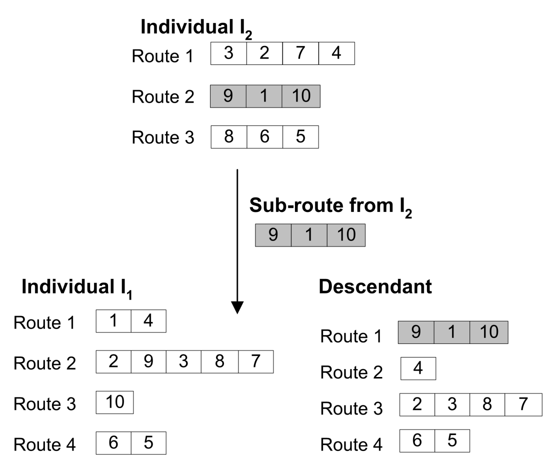

Figure 3: Example of the generic crossover.

Two categories of operators are considered: crossover and mutation. They must be able to deal with the two levels of the representation. Thus, they should be capable to change the delivery order within a specific route and to modify the allocation of demands to vehicles. In this last situation,

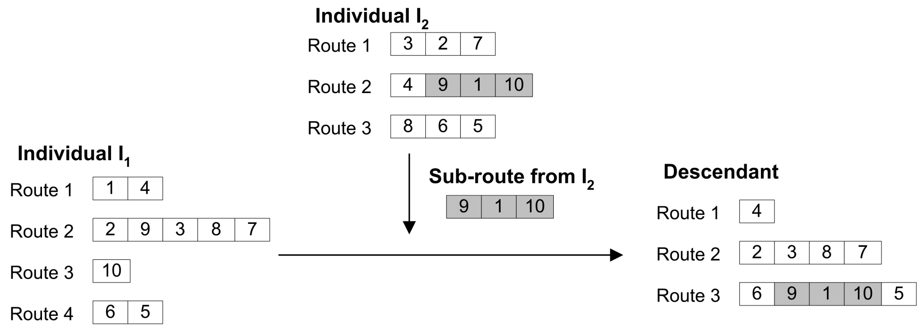

Figure 4: Example of the specific crossover.

they can, not only switch customers from one route to another, but also modify the number of vehicles belonging to a solution (adding and removing routes). Another important requirement is that the genetic operators must always generate legal solutions.

The crossover operator used in our approach does not promote a mutual exchange of genetic material between two parents. The crossover operates in the following way: when an individual from the selected set is submitted to this kind of operation, it receives a fragment of genetic material (more precisely, a route) from another parent and inserts it as its first route. After insertion, a repair process checks the original routes from the receiving individual and removes all customers that also appear in the inherited itinerary. This ensures that the new chromosome is legal, since there will be no repeated points. The donor is not modified. The example from figure 3 illustrates how crossover acts.

As an alternative to the generic crossover, we developed a more specific operator, sensitive to the geographical locations of customers. When accepting the fragment $\{a_1,...,a_n\}$ from the parent, the receiving individual determines which customer c is geographically closer to a_1. Then it inserts $\{a_1,...,a_n\}$ immediately after c.

The example from figure 4 helps to illustrate how this kind of crossover acts.

Descendants resulting from crossover can be subject to mutation. We consider four operators, based on proposals usually applied to order based representations:

- **Swap:** selects two customers and swaps them. Selected points can belong to the same or to different routes.
- **Inversion:** selects a sub-route and reverses the visiting order of the customers belonging to it.
- **Insertion:** selects a customer and inserts it in another place. The route where it is inserted is selected randomly. It is possible to create a new itinerary with this single customer, with probability $\frac{1}{2 \times V}$ (V represents the number of vehicles of the current solution).
- **Displacement:** selects a sub-route and inserts it in another place. This operator can perform intra or inter displacement. Just like in the previous operator, it is also possible to create a new route with the subsequence.

All genetic operators described have a specific probability of application to a single individual.

3.3 Path Representation

The standard EC algorithm used for comparison with the GVR approach is based on Path Representation (PR) [9] which is commonly used for order based problems. In this model, the chromosome can be seen as a single route, and the clustering of demands is determined at the interpretation phase, in positions where the vehicle capacity is exceeded. We used two different sets of operators with PR. The first is composed of common operators used for this kind of problems: Partially-Mapped Crossover and Swap mutation. The second set includes those operators used by GVR (the only difference is that they can't create new routes in a solution). The algorithm with PR and the GVR operators is referenced as the modified PR model.

4. EXPERIMENTAL RESULTS

To evaluate our approach we performed an extensive collection of tests with instances from some well known benchmarks. For the CVRP we used 12 instances from Augerat Set A (instances A32k5, A54k7, A60k9, A69k9, A80k10), Augerat Set B (instances B57k7, B63k10, B78k10) and C. and Eilon (instances E76k7, E76k8, E76k10, E76k14). For each instance of the datasets, the number of customers is given by the first number on the instance name. The main diference between these sets of problems is their tightness (the ratio between demand and capacity) and the location of customers.

For the VRPTW were used 12 instances from Solomon's benchmarks. We choose four instances from each one of the three different sets of problems, distinguished by the distribution adopted for customers. All sets consist of 100 customers plus the depot and are characterized by a short scheduling horizon, allowing only a few customers per route.

The settings of the EC algorithm are the following: Number of generations: 50000 (except for the instance A32k5, where only 10000 generations were required); Population size: 200; Tournament selection with tourney size: 5; Elitist strategy; CVRP Crossover rate: 0.75; VRPTW Crossover rate: {0.25, 0.5}; CVRP Mutation rates: swap: 0.05; inversion: 0.1, 0.15}; insertion: 0.05; displacement: 0.2; VRPTW Mutation rates: swap: 0.1; inversion: 0.1; insertion: 0.25; displacement: 0.25.

We performed an extended set of tests, with different probabilities of application for the genetic operators and verified that the ones selected for this paper are enhance the performance of the algorithm.

In VRPTW we used generic crossover. As for the CVRP, we used the specific version, since results achieved in some preliminaries tests showed that this second option enables the algorithm to discover better solutions (this improvement is not visible in VRPTW instances).

For every set of parameters we performed 30 runs with the same initial conditions. All initial populations were randomly generated. Statistical analysis was performed with a level of significance of 0.05.

4.1 CVRP

In table 4.1, we present, for all CVRP instances, the results achieved by GVR and PR. The table shows the best solutions found by the two algorithms as well as the averages of the best solution found in each of the 30 runs. The Column "Previous" indicates the best solution that were known when our research started. A brief perusal of the results reveals that GVR was able to find, for all instances, solutions with lower cost than those discovered by the PR model.

Examining the column with the averages, the values for GVR are also consistently better than the averages of the PR approach. These differences are statistically significant for every instance and for both settings. In fact, the averages obtained with GVR are close to the values of the best solutions. The distances range between 0.1% and 3.4%. As for the PR approach, these distances are superior (ranging between 19.1% and 40.9%). Another interesting point is that GVR was able to find new best solutions (instances A60k9, A69k9, B57k7, B78k10 and E76k14), whilst PR couldn't even reach any previous record.

This brief analysis shows that the introduction of a new representation with the associated genetic operators was able to improve the performance of a standard evolutionary algorithm.

The PR model had a poor performance. The question that arises at this point is: can the GVR genetic operators improve its performance? A first answer can be drawn by looking at table 4.1. It shows for the same instances and parameters, the comparison between GVR and the modified PR. Looking at the table we are able to realize that there is an improvement over the results achieved by the standard PR model. This algorithm, unlike the previous one, can discover good solutions in several instances. Nevertheless, unlike GVR, it didn't discover any new best solution.

A closer examination at the averages reveals that they have lower values than those obtained with PR. For all instances and settings, the differences between PR and the modified PR are statistically significant. On the contrary, the differences between GVR and modified PR are not statistically significant, with the exception of instances A32k5, B54k7, E76k14 (for both settings), and E76k7 (Inversion = 0.15). With modified PR, in most of the instances, the distances between the averages and the overall bests are in the same range of those achieved with GVR.

4.2 VRPTW

Table 4.2 presents the results obtained by GVR and PR. With VRPTW, like in the previous experiments, GVR also performs well: it can discover optimal solutions (instances C101 and C102) and solutions within a good range of the known best. When examining the results achieved with PR, it is clear that this algorithm is unsuitable for VRPTW. In all instances, results are very poor (both single bests and averages). Differences between results of these models are statistically significant.

In the CVRP variant, GVR genetic operators enabled PR to enhance its performance. The results of the modified PR model on the VRPTW are contained in table 4.2. A brief examination reveals that the genetic operators are unable to improve the performance of the algorithm to the same level of GVR. The modified PR algorithm didn't have the same degree of success in VRPTW, contrary to the CVRP. A more detailed analysis shows, nevertheless, some differences. Averages of the best solutions found by modified PR are slightly better than those achieved by the standard model. With the exception of R101, RC102, RC103 (both settings) and R104 (Crossover = 0.5), for all the other instances, differences are statistically significant. The GVR genetic operators were able to introduce some improvements on the results.

4.3 Comments

Results obtained with both variants of VRP show that GVR is an effective representation for this problem, providing a remarkable improvement over the performance exhibited by a standard PR approach.

A more detailed analysis reveals that, in the CVRP variant, the addition of an extended set of operators gives the PR model the ability to improve its performance and to obtain results that are similar to the ones achieved by GVR. This situation might be explained by the fact that it is easier to find good solutions for the CVRP than it is for VRPTW (since this last variant possesses additional time constraints). This way the advantage of GVR is not so evident (even though this was the only approach that was able to find new best solutions for several instances of CVRP). The structure of the instances used in the tests also helps modified PR to achieve good results. In all the examples used in the experiments, the optimal solution requires the load of the vehicles to be near its capacity. On the other hand, in situations where good solutions require a load considerably below this value, PR will be unsuitable and GVR might introduce a significant improvement. Another important conclusion to draw from the results is that it seems to be advantageous to use a wider set of genetic operators, since it might help to maintain diversity during search phase. It was this modification that enabled modified PR to obtain good results in the CVRP.

As for the VRPTW, only GVR was able to achieve good results. This clearly shows that in complex VRP problems, adopting a good representational scheme is essential and that the choice of a representation that is sensitive to the structure of the solutions provides an important advantage.

5. CONCLUSION

In this paper, a systematic comparison between GVR, a representational scheme designed to deal with the information required for VRP solutions, and a standard order based EC algorithm was made.

Results show that representation plays an important role in routing problems. GVR is consistently better at discovering good solutions than evolutionary techniques based on PR models. Moreover, the advantage provided by GVR over these standard representations is more evident as the complexity of the instances being solved increases. When we tried to find good solutions to the VRPTW variant, the use of GVR proved to be essential. We also verified that the use

Table 1: **Results from GVR and PR model (CVRP).**

		GVR				PR			
		Inv. = 0.1		Inv. = 0.15		Swap = 0.1		Swap = 0.15	
Instances	Previous	Best	Avg.	Best	Avg.	Best	Avg.	Best	Avg.
A32k5	784	784	790.0	784	786.3	857	973.53	879	990.83
A54k7	1167	1167	1186.2	1167	1188.9	1303	1459.93	1330	1476.50
A60k9	1358	1354	1377.9	1358	1372.4	1557	1666.73	1524	1641.57
A69k9	1167	1165	1182.0	1165	1179.9	1455	1578.47	1486	1576.17
A80k10	1764	1777	1813.8	1774	1811.0	2064	2217.13	2043	2214.53
B57k7	1153	1140	1141.2	1140	1140.7	1376	1549.07	1292	1531.13
B63k10	1496	1496	1544.0	1497	1545.7	1709	1896.00	1707	1859.73
B78k10	1266	1223	1254.6	1221	1252.3	1418	1661.33	1473	1655.23
E76k7	682	687	704.8	683	705.0	861	952.40	834	960.73
E76k8	735	738	755.3	740	755.4	897	1003.50	922	999.30
E76k10	832	841	856.6	837	854.0	1005	1091.00	976	1080.10
E76k14	1032	1022	1043.5	1030	1043.0	1138	1217.43	1140	1232.03

Table 2: Results from GVR and modified PR (CVRP).

		GVR				Modified PR			
		Inv. = 0.1		Inv. = 0.15		Inv. = 0.1		Inv. = 0.15	
Instances	Previous	Best	Avg.	Best	Avg.	Best	Avg.	Best	Avg.
A32k5	784	784	790.0	784	786.3	796	807.2	796	805.2
A54k7	1167	1167	1186.2	1167	1188.9	1167	1184.2	1167	1182.4
A60k9	1358	1354	1377.9	1358	1372.4	1358	1371.0	1358	1374.2
A69k9	1167	1165	1182.0	1165	1179.9	1168	1184.6	1166	1183.0
A80k10	1764	1777	1813.8	1774	1811.0	1776	1807.6	1780	1813.4
B57k7	1153	1140	1141.2	1140	1140.7	1145	1148.7	1145	1147.9
B63k10	1496	1496	1544.0	1497	1545.7	1510	1542.9	1520	1545.9
B78k10	1266	1223	1254.6	1221	1252.3	1223	1247.8	1228	1256.7
E76k7	682	687	704.8	683	705.0	682	704.0	691	711.3
E76k8	735	738	755.3	740	755.4	740	755.0	739	756.5
E76k10	832	841	856.6	837	854.0	837	855.5	840	855.0
E76k14	1032	1022	1043.5	1030	1043.0	1032	1046.3	1032	1045.8

Table 3: Results from GVR and PR model (VRPTW).

		GVR				PR			
		Cx = 0.25		Cx = 0.5		Cx = 0.25		Cx = 0.5	
Instances	Optimal	Best	Avg.	Best	Avg.	Best	Avg.	Best	Avg.
C101	827.3	827.3	832.63	827.3	845.18	3387.4	3798.90	3490.6	3823.47
C102	827.3	827.3	866.24	827.3	869.44	2965.5	3378.50	3055.4	3398.39
C103	826.3	826.3	900.49	826.3	909.42	2577.0	3048.66	2812.8	3051.49
C104	822.9	854.8	916.82	834.7	941.96	2313.0	2819.15	2378.1	2791.36
R101	1637.7	1683.4	1722.08	1666.3	1723.70	4521.8	4616.18	4523.6	4618.11
R102	1466.6	1477.8	1538.89	1486.1	1536.98	4295.7	4470.06	4277.8	4477.48
R103	1208.7	1244.0	1300.86	1267.7	1321.56	4249.2	4392.88	4252.7	4381.22
R104	n/a	1014.3	1071.29	1044.3	1085.37	4194.6	4321.86	4167.5	4303.11
RC101	1619.8	1671.2	1769.07	1686.5	1770.28	5898.0	6118.46	5882.3	6092.73
RC102	1457.4	1519.6	1612.80	1536.3	1611.79	5672.7	5914.81	5644.9	5945.70
RC103	1258.0	1379.4	1423.77	1353.8	1415.48	5560.8	5778.33	5566.4	5831.97
RC104	n/a	1205.5	1273.40	1179.9	1278.32	5435.6	5716.03	5416.5	5695.99

Table 4: Results from GVR and modified PR (VRPTW).

		GVR				Modified PR			
		Cx = 0.25		Cx = 0.5		Cx = 0.25		Cx = 0.5	
Instances	Optimal	Best	Avg.	Best	Avg.	Best	Avg.	Best	Avg.
C101	827.3	827.3	832.63	827.3	845.18	2912.2	3175.24	2933.4	3151.06
C102	827.3	827.3	866.24	827.3	869.44	2435.3	2725.22	2306.3	2669.69
C103	826.3	826.3	900.49	826.3	909.42	2076.4	2400.49	2057.5	2342.31
C104	822.9	854.8	916.82	834.7	941.96	1674.9	2177.07	1887.1	2192.23
R101	1637.7	1683.4	1722.08	1666.3	1723.70	4516.8	4609.60	4545.8	4612.53
R102	1466.6	1477.8	1538.89	1486.1	1536.98	4303.7	4405.79	4295.6	4420.06
R103	1208.7	1244.0	1300.86	1267.7	1321.56	4210.9	4316.03	4184.3	4335.40
R104	n/a	1014.3	1071.29	1044.3	1085.37	4179.8	4280.16	4181.4	4270.62
RC101	1619.8	1671.2	1769.07	1686.5	1770.28	5899.4	6002.20	5899.4	6013.89
RC102	1457.4	1519.6	1612.80	1536.3	1611.79	5736.7	5824.64	5736.9	5873.12
RC103	1258.0	1379.4	1423.77	1353.8	1415.48	5601.6	5729.54	5605.1	5760.62
RC104	n/a	1205.5	1273.40	1179.9	1278.32	5528.1	5676.52	5522.8	5702.27

of a diverse set of genetic operators helps EC algorithms to improve its search performance.

As future work, we intend to perform a detailed study on the importance of the genetic operators presented in this paper, to determine their specific role on the evolutionary process.

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